Swami Ramanand Teerth Marathwada University, Nanded, Maharashtra, India

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"Study Of Prefunctions Using Some Integral Transforms and Their Applications"

Research Proposal Submitted

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In the subject

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Submitted by

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1. Introduction to proposed research topic:

The differential equations have played a central role in every aspect of applied mathematics form very long time and with the advent of the computer, their importance has increased father. Thus investigation and analysis of differential equations cruising in applications led to many deep mathematical problems; therefore, there are so many different techniques in order to solve differential equations. In order to solve differential equations, and integral equations, the integral transforms were extensively used and thus there are several words on the theory and application of integral transforms.

In mathematical sciences a transformation is such device which is useful for the conversion of one function f(t) into another function of new variable $\mathcal{F}(s)$. The integral Transform of a function$f(x)$ in [a, b], is denoted by $\mathcal{F}(s) = \phi\{f(x)\}$ and defined by

$$\mathcal{F}(s) = \phi\{f(x)\} = \int_a^b K(x,s)f(x)dx.$$

Form almost two centuries we have used integral transforms successfully in Mathematical science and Engineering field. Integral transform methods providing unifying mathematical approach to the study of electrical, network, devices for energy conversion and control antennas and the component of electrical system. Integral transforms applied equally to the study of electrical communication by wire or optical fibers, to wireless radio propogation. Integral transforms provide both theory and applications for science and engineering. Integral transform methods are at the heart of engineering curriculum. In spite of all that things their uses are still

predominant in advanced study and research. So these integral transforms are attracting to all researchers, since their marvelous work and applications in almost all fields.

We have the following definitions for further use

(a) Laplace Transform: The Laplace Transform is defined as follow

$$\mathcal{L}[f(t)] = \mathcal{F}(s) = \int_0^\infty e^{-st} f(t)dt\,, \qquad t > 0$$

(b) Prefunctions and Extended Prefunctions:

The pre-exponential functions is denoted by $pexp(t, \alpha)$ and is defined as,

$$pexp(t, \alpha) = 1 + \sum_{n=1}^{\infty} \frac{t^{n+\alpha}}{\Gamma(n+1+\alpha)}, t \in \mathbb{R} \text{ and } \alpha \geq 0$$

The Pre-cosine function and Pre-Sine function is denoteded by $pcos(t, \alpha)$, $psin(t, \alpha)$ respectively and is defined as

$$pcos(t, \alpha) = 1 + \sum_{n=1}^{\infty} (-1)^n \frac{t^{2n+\alpha}}{\Gamma(2n+1+\alpha)},\ psin(t, \alpha) = \sum_{n=0}^{\infty} (-1)^n \frac{t^{2n+1+\alpha}}{\Gamma(2n+2+\alpha)}, t \in \mathbb{R}$$

The Extended Prefunction is defined as

$$pM_{32}(t, \alpha) = \sum_{n=0}^{\infty} (-1)^n \frac{t^{3n+2+\alpha}}{\Gamma(3n+3+\alpha)}, t \in \mathbb{R}$$

2. Review of Literature and Development in the relevant subject (Previous work done in the relevant areas):

Integral transform method is an efficient method to solve differential equations, system of differential equations, integral equations, system of integral equations and so on. Many researchers have interest in the study of integral transform and their applications in field of

Engineering and other. The oldest and notable transform also most commonly used is the Laplace transform developed by P.S. Laplace in 1780 (2) and Fourier transform by J. Fourier in 1822 (3). The only difference between the Laplace transform and Fourier transform is the Laplace transform can be defined for both stable and unstable system while the Fourier transform can only defined on a stable system. Other available transforms ate Stieltjes transform (4), Mellin transform (5-6), Hankel transform (7), Hilbert transform (8) Radon transform (9), Lagurre transform (6) and Wavelet transform (11). More recently G.K. Watugula introduced the Sumudu transform in 1993 (12) to solve differential equations and control engineering problems. Khan and Khan introduced the Natural transform in 2008 (13), T. M. Elzaki given the Elzaki transform in 2011 (14), K.S. Abooodh has developed Aboodh transform in 2013 (1) which is used to solve both ordinary and partial differential equations. Recently in 2015 Srivastava et al introduced M – transform (15). Also many researchers studied deeply in the comparison between two transforms and their properties (15-16). Also Khandeparkar, Deo and Dhaigude, have defined new kind of Exponential, Trigonometric and hyperbolic functions, they are coined as Prefunctions and extended Prefunctions in 2010(17-19). It is shown that these pre-functions and extended Prefunctions satisfy many interesting properties and relations. Some properties and Laplace Transforms of these functions are studied and solutions of initial value problems for non-homogeneous linear ordinary differential equations,Voltera integral equations, Voltera integro-differential equations obtained (20-22). A. p. Hiwarekar studied relation of perfunctions and Extension of Prefunctions and its relation with Mittag- Leffler function in 2017(23).

3. Objectives of the Research:

- To study Integral Transforms like Fourier Transform, Laplace Transform, Sumudu Transform, Elzaki Transform etc., and its applications.
- To study ordinary and differential equations.
- To study of applications of some new integral transforms.
- To study basic concept of Prefunctions and extended Prefunctions.
- To study Integral Transform of Prefunctions and Extended Prefunctions.
- To obtain Prefunctions as solutions of differential equations, integral equations.
- To publish our work in reputed journals.

5. Hypothesis:

The proposed work is to prove that how mathematics is useful in the other branches of science. We hope that our research work will be useful in applied science and engineering field. Hence, in the present work, we will try to show that Prefunctions and extended Prefunctions are solutions of differential and integral equations. Also show that they are solutions of intrgo-differential equations.

4. Methodology to be adopted:

- To refer the relevant reference books on integral transform
- Collection of references through research journals/internet.
- By studying references finding new results.
- Finding out applications of new results with supportive examples.

- Understanding the knowledge about current research topic by participating in the topic related conferences, workshops, seminars.

5. Importance of Study:

The study of integral transform plays an important role because the study finds applications in many important areas as diverse as differential equations, integral equations, mathematical economics, and engineering.

a) International Status:

Researchers and scientists have contributed in the development of theory of Integral transform like Laplace, Fourier, Mellin, Hilbert, Radon and wavelet. More recently, some new integral transform introduced by Watugula, Khan,Elzaki, Aboodh, Kashuri, Fundo and Srivastava et al, Mohand and Mahgoub, Sadik etc

b) National Status:

Indian's also contributed remarkably in the development integral transform like Senthil Kumar, Naresh, Umamaheshwar Rao, Eswarlal, Vasuki, Reddy, Khandeparkar, Deo, and Dhaigude. Latpate , Ingle, Shaikh, Hiwarekar, Sontake, Kamble, Sawant and many more.

d) Scope of Study:

From last two centuries, integral transforms have been applied almost every field of science, engineering, biology, physics and mathematics. Many applications of integral transform can be found in fluid dynamics, stochastic dynamical system, plasma physics, and control theory.

6. Proposed Work plan / Formation and Structure of study:

First Year: At this stage we will collect the required information by means of journals, books and some research papers related to research topics. We would like to visit some libraries and universities. In this year we will complete the course work. Progress report will be submitted at the end of this year.

Second Year: At this stage we will study the basic properties of Laplace Transform, Sumudu Transform, Elzaki Transform and New Integral Transform.Also we will study Prefunctions and Extended Prefuncions. The collected material will be proceeded and attempt will made to discover the finding in the direction of research different methodology, tools, techniques and guidance will be apply for new research problems. To the study of new research problem in different form and it will be submitted to research journals for publication.

Third Year: The outcome of our research work will be presented in national and international gatherings such as, conferences, seminars, workshops during this year. The research work will be published in the several reputed journals. The printing and binding of project work. The papers of project work based on existence theory will be communicated to the different national and international journals for the publication. Two progress reports will be submitted at the end of this year and submission of final synopsis will be done in this year to the authority.

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